CS 5860

Thur Oct 6, 2011 Primitive Recursive Arithmetic

Examples of primitive recursion

$a_0(x,y) = x+1$ $\quad S(x) = x+1$ so $a_0(x,y) = S(x)$

$a_1(x,y) = add(x,y)$

$$\boxed{\begin{cases} add(0,y) = y \\ add(S(x),y) = S(add(x,y)) \end{cases}}$$

note $S(add(x,y)) = a_0(a_1(x,y),y)$

$a_2(x,y) = mult(x,y)$

$$\boxed{\begin{cases} mult(0,y) = 0 \\ mult(S(x),y) = add(mult(x,y),y) \end{cases}}$$

i.e. $(x+1)\cdot y = x\cdot y + y$

note $add(mult(x,y),y) = a_1(a_2(x,y),y)$

$a_3(x,y) = exp(x,y)$

$$\boxed{\begin{cases} exp(0,y) = 1 \quad \text{i.e. } y^0 = 1 \\ exp(S(x),y) = mult(exp(x,y),y) \end{cases}}$$

$y^{x+1} = y^x \cdot y$

note $mult(exp(x,y),y) = a_2(a_3(x,y),y)$

$a_4(x,y) = hypexp(x,y)$

$$\boxed{\begin{cases} hypexp(0,y) = y \\ hypexp(S(x),y) = exp(hypexp(x,y),y) \end{cases}}$$

note $exp(hypexp(x,y),y) = a_3(a_4(x,y),y)$

$a_{n+1}(x,y)$

$$\boxed{\begin{cases} a_{n+1}(0,y) = y \\ a_{n+1}(S(x),y) = a_n(a_{n+1}(x,y),y) \end{cases}}$$

Can think of $a_n(x,y)$ as a function of n, x, y.

This is Ackerman's function in one form.

It is not primitive recursive.